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香 港 大 學

THE UNIVERSITY OF HONG KONG

Bachelor of Engineering

Department of Electrical and Electronic Engineering

ELEC 3703 Queuing Theory

Examination

Date: December 14, 2009 Time: 2:30 p.m. - 4:30 p.m.

Q.	*Points*
1	
2	
3	
4	
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Total

Answer *ALL* questions. ***You must submit this question paper.***

Write your answers on the question paper.

Do *NOT* use a separate answer book.

In each question, space is provided for your answer. Your answer should be concise and to the point.

The amount of space provided in each question does *NOT* necessarily reflect the amount of information required in your answer.

State clearly any assumptions that you have made.

Use of Electronic Calculators:

For papers which permit the use of calculators, electronic calculators, including programmable calculators, may be used provide that the calculators are battery-powered, silent in operation and with neither print-out nor graphic/word-display facilities and do not use dot-matrix technology in the main display.

Only approved calculators as announced by the Examinations Secretary can be used in this examination. It is candidates' responsibility to ensure that their calculator operates satisfactorily, and candidates must record the name and type of the calculator used on the front page of the examination script.

1. (10 points) The probability density function of a *non-negative* random variable X, is defined as:

$$f_X(x) = \alpha_1 \mu_1 e^{-\mu_1 x} + \alpha_2 \mu_2 e^{-\mu_2 x}, \text{ where } \alpha_1 + \alpha_2 = 1,\ \alpha_1 > 0,\ \alpha_2 > 0,\ \mu_1 > 0,\ \mu_2 > 0,\ x \ge 0.$$

(a) (4 points) **Derive** $F_X(s)$, the Laplace transform of the probability density function of X.

(b) (6 points) By using the result of part (a), **compute** the mean and variance of the random variable X.

2. (20 points) An absent-minded salesperson schedules two client appointments for the same time. The appointment durations are *independent* and *exponentially* distributed with mean 50 minutes. The first client arrives on time, but the second client arrives 10 minutes late. **Compute** the expected time between the arrival of the first client and the departure of the second client. **Give** your answer in 3 decimal places.

3. (20 points) Suppose that $X_1, X_2, \ldots, X_N$ are *non-negative, independent,* and *identically-distributed* random variables, each of which has the *same* probability density function $f_X(x)$, where $x \geq 0$. Denote by $X^*(s)$ the Laplace transform of $f_X(x)$. N is a *non-negative* discrete random variable. The probability generating function of N is $G(z)$. Define $Y = \sum_{k=1}^{N} X_k$.

   (a) (6 points) **Show** that $Y^*(s) = G(X^*(s))$, where $Y^*(s)$ is the Laplace transform of $f_Y(y)$, the probability density function of Y.

   (b) (6 points) The probability mass function of N is defined as $P(N = n) = p \cdot (1-p)^{n-1}$, where $P(N = 0) = 0$ and $n = 1, 2, 3, \ldots$. **Derive** $G(z)$.

(c) (8 points) Let $f_X(x) = \lambda e^{-\lambda x}$ and $P(N = n) = p \cdot (1-p)^{n-1}$, where $P(N = 0) = 0$, $x \geq 0$, and $n = 1, 2, 3, \ldots$. By using the results of parts (a) and (b), **compute** $Y^*(s)$ and $f_Y(y)$, where $y \geq 0$.

4. (30 points) Consider a birth-death process with the following birth and death coefficients:

$$\lambda_k = (k + 2)\lambda \qquad k = 0, 1, 2, \ldots$$
$$\mu_k = k\mu \qquad k = 1, 2, 3, \ldots$$

All other coefficients are zero. Let p_k be the limiting probability that the system contains k members.

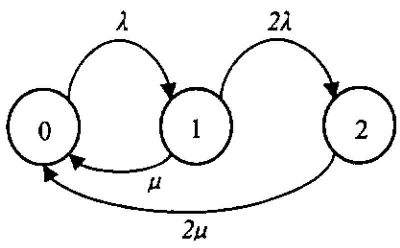
(a) (3 points) **Draw** the state-transition-rate diagram of the Markov chain for the system.

(b) (6 points) **Express** p_k in terms of p_0, where $k = 1, 2, 3, \ldots$.

(c) (9 points) **Express** p_k in terms of k, λ, and μ only, where $k = 0, 1, 2, \ldots$

(d) (12 points) **Find** $\overline{N}$, the expected number of members in the system.

5. (20 points) Consider the Markovian queueing system shown below. Branch labels are birth and death rates. Node labels give the number of customers in the system.



(a) (8 points) **Solve** for p_k, the limiting probability that the system contains k customers, where $k = 0, 1, 2$.

(b) (3 points) **Calculate** ρ, the probability of a busy system.

(c) (3 points) **Compute** $\overline{N}$, the expected number of customers in the system.

(d) (6 points) **Give** the transition rate matrix **Q** for the queueing system and the matrix equation relating **Q** to the probabilities found in part (a).

***** END OF PAPER *****